**Question 8.** A satellite orbits the Earth at a distance of 5×106m from the Earth’s surface. What is its speed?



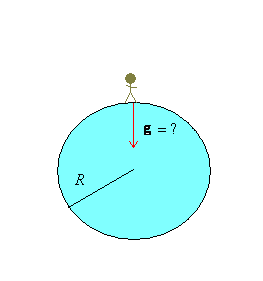
**Question 6**. What should be the radius of a satellite purposed to orbit the Earth twice a day. Use ME = 6×1024kg.

Combining N2L in centripetal direction with gravitational force equation:



**Example: Gravitational field near the Earth’s surface**

What is the gravitational field of the Earth, near its surface?



Well, it is



That is, it is just the acceleration due to gravity. This isn’t a coincidence. The gravitational field at a point gives the acceleration due to gravity at that point.

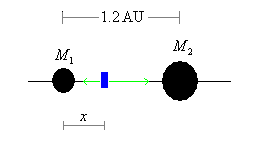
**Example: Moon’s gravitational field**

What is the acceleration due to gravity at the surface of the moon? This is just the magnitude of **g** at the Moon’s surface, which is:



**Example: Avoiding two black holes**

Suppose there are two black hoes 1.2AU’s apart. One has a mass, M1 equal to the Sun, and the other, M2 has a mass twice that of the Sun. Not wanting to die, where should we position our spaceship so that the net gravitational force of two black holes cancels?



In order to feel no force at that point, we need the net gravitational field, **g**, to be 0 there. So let’s construct **g** at an arbitrary point inbetween the two black holes.



and see where it is equal to **g** = 0,



Let’s choose the + sign first. This gives us:



Filling in the values we get,



So we should position our spaceship 0.5 AU away from the left hand black hole.

**Example: Gravitational force of attraction between Sun and Earth**

With what force does the Sun attract the Earth, given MS = 2×1030 kg, ME = 6×1024 kg,

and are r = 1.5×1011m apart.

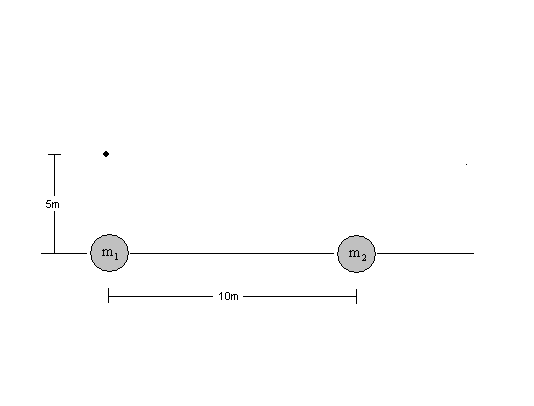
The force of attraction would be,



With what force does the Earth attract the Sun? By N3L, and explicitly from the formula above, the force is the same as that with which the Sun attracts the Earth, 3.56×1022 N.

**Example**

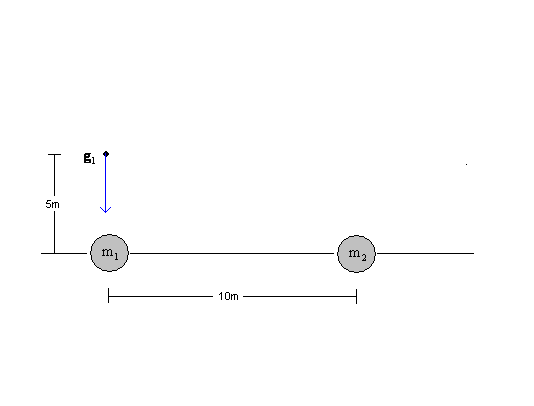
Suppose have two masses m1 = 6×109kg, and m2 = 8×109kg at the indicated positions. What is field 5m above m1? What is the acceleration due to gravity at that point? Well have,



First we calculate the field due to m1. From before we have,



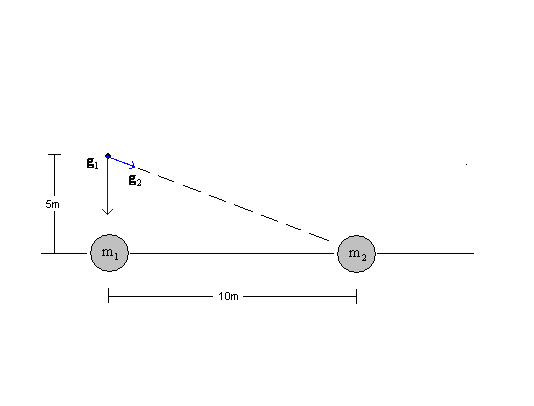
Drawing this in we get,



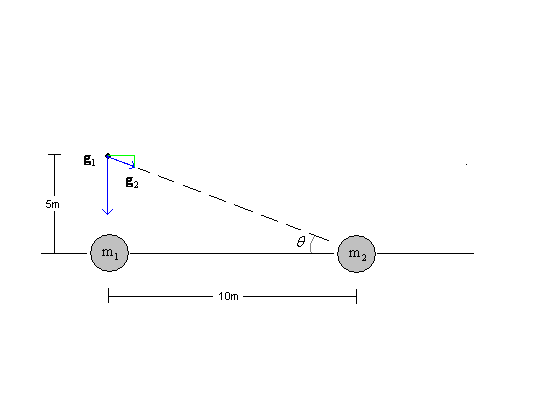
while, (ask for direction) using the Pythagorean theorem, **g**2 is:



and so we have,



Now add the two **g** vectors together. At this point it is probably best to convert **g**2 to the unit basis. This can be done by drawing the vector out (already done I imagine), and finding its components along the axes. Then the addition can proceed as usual by vector addition. Can put in polar form if want. So I would break **g**2 into its components,



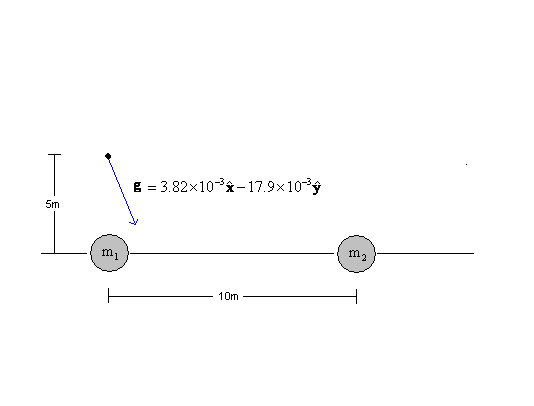
The grey-angle is θ = tan-1(5/10) = 26.6 degrees. Therefore, the angle inside the green triangle is also 26.6 degrees. And we have,



And therefore the net gravitational field at the point is:



illustrated below,



the magnitude and direction of the field is:



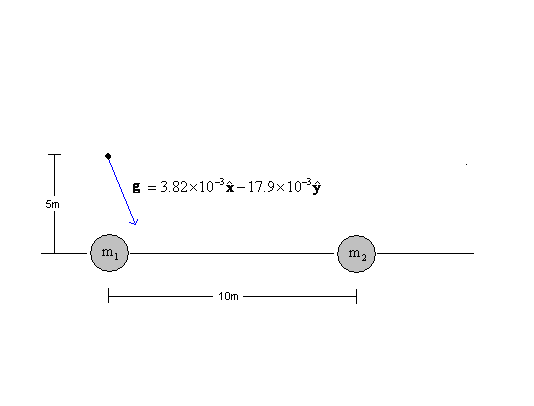
and so,



And so the acceleration due to gravity at that point would be **a**g = 0.0183 m/s2 in the specified direction.

**Example: Gravitational force due to the two spherical masses**

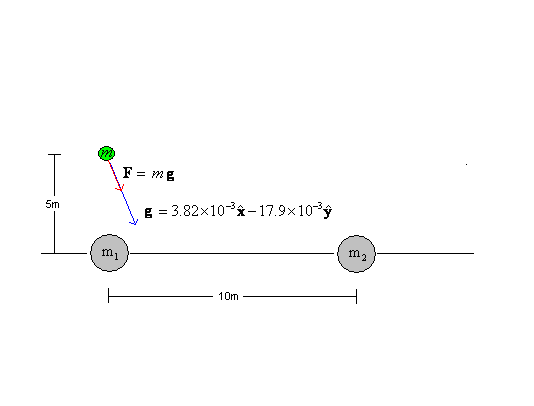
Consider again the two spherical charges examined above, illustrated below,



where the magnitude and direction of the field was found to be:



Now here’s the question. If we place a mass (m = 12kg) at the point 5 meters above m1, what force will it experience.

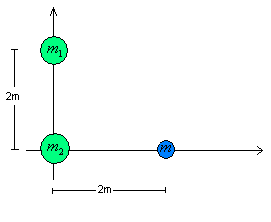


We would first calculate **g** at that location (already did). And then use **F** = m**g**. So,



**Problem 10**

Consider the following set up. Suppose m1 = 1 ×109 kg, while m2 = 2 × 109 kg. What is the magnitude of the force that m = 30kg will feel?



The net field is just the sum of the two individual fields. This is:



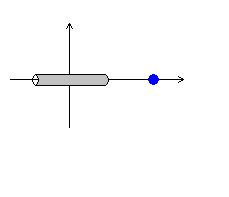
so the magnitude of the field is:



and so the force on the mass is:



8. Suppose a cylindrical mass (M = 90 billion kg, and L = 14km) sits with its center at the origin. And another mass, m = 12 billion kg sits a distance 18km from the origin. What will be the acceleration of m towards M?



We must calculate the gravitational field that the cylindrical mass sets up at the point d = 18km. The gravitational field set up a little piece of mass at coordinate x in the cylinder is



where ρ = M/L. Adding up all fields from each mass gives,



So we get:



**Example: Orbital radius of planet X.**

Suppose an astronomer locates another planet, planet X, in our solar system. Suppose that she calculates (by observing the motion of the planet) that it would take 25 Earth years for planet X to make one complete revolution about the Sun. How far away is planet X from the Sun (in AU’s)?

We can use Kepler’s third law above to make this calculation. Solving for r we have,



An AU (Astronomical Unit) is the distance, rE, between the Earth and Sun. 1 AU = 1.5×1011m. We can calculate how far away planet X is in AU’s by dividing by this distance. But we can do this directly without knowing what an AU is in meters. We would do this in the following way,



So rX is 8.55 AU. What is the orbital speed of the planet? We can use?



**Example: Geosynchronous orbit**

At what height above the Earth’s surface would a spy satellite have to be launched in order to put it in geosynchronous orbit?

Geosynchronous orbit is an orbit which rotates around the Earth at the same rate that the Earth rotates about its own axis. Thus a satellite in such an orbit will contually remain above the same spot of the Earth. Therefore its angular velocity is:



Now let’s put v in terms of ω in the equation relating v and r.



Filling in our particular ω, we have,



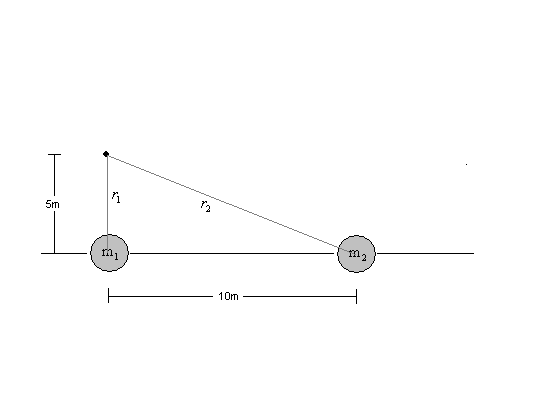
This is a height,



above the Earth’s surface. This is about 6 times the radius of the Earth – so its pretty far out there.

**Example**:

Suppose have two masses m1 = 6×109kg, and m2 = 12×109kg at the indicated positions. What is net potential 5m above m1?



Well have,



What is the gravitational potential at the point 1m to the right of m1? Then we have,



**Example**

What is the potential energy of a block of mass m, a relatively short distance y above the surface of the Earth?

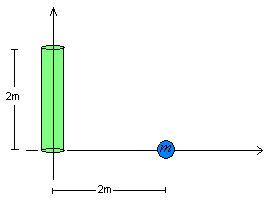
The potential energy mV, and we may use the expression V = -grr = -gyy (because we’re going in the y-direction) since we’re close to the surface of the Earth and so **g** is roughly constant there.



This is just our familiar expression from before as you’ll recall.

**Problem 11**

What is the gravitational potential energy of the mass, m = 200kg, if it is situated next to a uniform cylinder with mass M = 3×109kg as shown below?



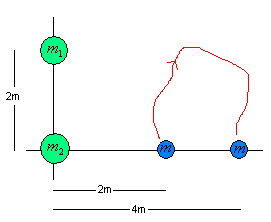
The potential at that point is:



and so the potential energy is:



**Problem** 6. Given the m1 = 5 ×109 kg, and m2 = 2 × 109 kg. How much work would be required to move the mass m = 150kg along the following path.

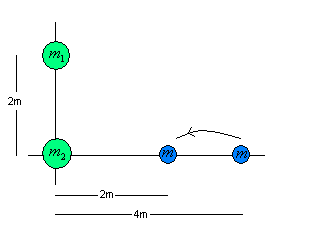


The work required would simply be, from the WE equation, the change in potential energy associated with moving the mass from the initial to final location,



**Example: Work required to move mass m around in mass configuration.**

Suppose we have m1 = 9×109kg and m2 = 4×109kg on the y axis as shown.



Determine work required to move bluey (m = 12kg) from far away to closer (again not changing its KE). We would have,



**Example: Speeding the mass up**

Suppose we want to move the mass the other way: from 2m to 4m. But we also want to speed it up in the process so that it goes from rest to a velocity of 75cm/s? What work is required?

Well, using the WE equation,

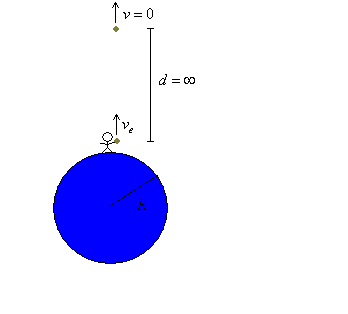


which comes to:



**Example: Escape velocity**

If you throw a rock up in the air, it will eventually come back down. If we throw it faster it will take longer to come down. Is there is velocity with which we can throw the rock so that it won’t come back?



The way to formulate this problem is as follows. As we throw the rock faster, the distance above the Earth that it takes for it to start coming back (i.e. for its velocity to drop to 0 and then go negative) becomes greater and greater. In order to specify that the rock never comes back, we would specify that the distance where it begins to come back, i.e., when its velocity is 0 is infinite. Siince it can never reach infinity, it can never come back. Then we use the work energy equation on the rock between its initial and final positions.



Now you are doing no work on the object between its release at ve, and when it ‘comes back’ at r = ∞. Therefore we have,



Solving for ve we have,



Filling in the values for Earth, we get,



So Earth’s escape velocity is **11.2 km/s**. This is pretty fast. This one reason why NASA launches its rockets from Florida. Florida is closer to the equator than the other states. As such, as the planet rotates about its own axis, Florida is going faster than the other states because its radius of revolution is larger than the other states. For instance the North and South pole isn’t rotating at all and so has no angular velocity, and therefore no tangential speed at all. But the more you move towards the equator, the larger the radius of rotation becomes, and therefore the larger the tangential velocity becomes. Therefore by launching at Florida, NASA doesn’t have to come with all of the energy for the 11.2 km/s by itself.

7. Suppose you take the sun, M = 2×1030kg, and shrink it down to a radius R. At what R would the escape velocity from the surface of the Sun be equal to the speed of light?

The escape velocity is determinable from the WE equation. We look for the velocity an object (in this case a photon) is required to have on the surface of a mass, M, so that it will slow down to v = 0 by a distance of r = ∞ away. Since it is making the trip under the influence of gravity alone, no non-conservative work is done on it.



Now we look for what R is required to have vesc = c,



Problem. If a satellite orbits Earth (M = 5.98×1024kg) at a radius of 5×107m, what is its speed?

Using N2L in the centripetal direction, we have, considering the gravitational force acting on the satellite…



where M is the mass of Earth, m that of the satellite, and r the orbital radius of the satellite. Solving for v…



Plugging the numbers in…



**Example: Total mechanical energy of Earth in rotation around Sun**

What is the total energy possessed by the Earth? It has gravitational potential energy due to its proximity to the Sun, and it also possess kinetic energy because it is moving. Let’s calculate its total energy then. In the process we’ll discover an important formula for the energy of an object orbiting another.

The earth orbits the Sun at a radius of R = 1.5×1011m. The mass of the Sun is M = 2×1030kg, while the mass of the Earth is m = 6×1024kg. Therefore its potential energy is:



What is the Earth’s kinetic energy? Well, we’ll recall that for an orbiting body, we found that:



and so KE is:



Observe how this is just ½ the PE! The total energy therefore is:



which is negative. But there is nothing peculiar about the negative sign. The negative sign is an indication that the planet is *bound* to the Sun. If its total energy were positive, then this would mean that the planet is not bound to the Sun. In any event, we have an important formula for the total energy of a body in orbit about another,



The first two parts of this expression hold for the electric force too. The total mechanical energy of an electron orbiting a proton is just ½ of its potential energy.

9. Certain neutron stars (extremely dense stars) are believed to be rotating at about 2 rev/s. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during the rapid rotation?

We must use N2L:



10. Planet X, with a mass of 5.9 × 1024 kg and a radius of 1960 km, gravitationally attracts a meteorite that is initially at rest relative to the planet, at a great enough distance to take as infinite. The meteorite falls toward the planet. Assuming the planet is airless, find the speed of the meteorite when it reaches the planet's surface.

We must use the WE equation,



**Problem 12**

The Moon orbits the Earth approximately once every 30 days. If the mass of the Earth is 5.98×1024kg, how far away from the Earth is the Moon?

Using N2L and the law of gravitation we have:



And we know the period is 30days. So solving for r in that case:



**Question 10.** How fast would you have to throw a baseball at the Earth’s surface to have it circle the Earth and hit you in the head? You can take the radius of the Earth to be R = 6.36×106 m, and mass of Earth to be mE = 5.98×1024 kg.

We have:

